Investigating road traffic flows using stochastic automaton models

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The aim of this project was to study road traffic flows for different scenarios. Using cellular automaton models, the flow of buses, flow at traffic lights and flow at a roundabout were investigated. In the bus model it was discovered that clumping occurred at all densities and is linked to the frequency of buses on the road and the arrival and departure rates of passengers. In the traffic light code, the interval was optimised between red and green lights and the best values of $\omega = 0.2$, $t_{on} = 0.6$ for a single traffic light and $\omega = 0.3$, $t_{on} = 0.7$ for two traffic lights were found (where ω and t_{on} are parameters which determine the on/off period of the traffic light(s)). In the roundabout model the difference between a traffic light and roundabout was discussed and the flow rates at both low and high densities were compared between the two to decide if a roundabout is a suitable replacement for a traffic light. The Nagel-Schreckenberg (NaSch) model is the basis of the models used and is discussed in detail; the circular-road arrangement is used extensively throughout.

1. Introduction

Traffic flows play a significant role in each of our lives in the modern world. We are surrounded by examples, from pedestrian flows in busy subway stations to air-traffic flow at airports. In the 21st Century, with an unprecedented global population, the demand for mobility has never been greater. In industrialised countries the usual reaction to this increasing demand has been to expand the transportation infrastructure. Until recently, traditional planning methods have been sufficient. Notwithstanding, developed societies such as the UK are seeing the limitations of this expansion with road network capacities becoming saturated or in extreme cases exceeded. In some densely populated areas it can be financially or socially impractical to continue expanding road networks. Consequently, it is crucial that planning strategies are improved and existing transport networks made more efficient [1, 2].

Attempts to simulate traffic flow using fluid-dynamical models date back to the 1950s. One notable example being a theory of traffic flow using kinematic waves introduced by Sir James Lighthill, FRS, and Gerald Whitham at the University of Manchester in 1955 [3]. In subsequent years, methods of nonlinear dynamics were also used. However, owing to their computational simplicity, lattice gas automata [4] were successfully applied to simulate fluids and traffic flows. A significant step forward was the invention of a cellular automaton (CA) model based on Rule 184 (see Appendix B) introduced by Kai Nagel and Michael Schreckenberg in 1992 [5]. The Nagel-Schreckenberg (NaSch) model was the first of its kind to account for imperfect human behaviour and consequently explained the spontaneous formation of traffic jams. This is a key feature in modelling traffic networks. Using a suitable model, realistic predictions can be made for the development of traffic situations in the real world, which can be used to further improve the efficiency of road networks.

The NaSch model is used extensively in our investigations. The outline of this report is as follows: Section 2 introduces the classic one-dimensional NaSch model. Then the flow of buses is investigated, followed by the effects of traffic lights, and finally flow at a roundabout (Sect. 3). Our conclusion is given in section 4.

2. The Nagel-Schreckenberg Model

2.1. The model

The classic NaSch model is a probabilistic cellular automaton. It consists of a circular one-lane-road with discrete positions (cells) of length L. This one-dimensional ring contains L sites and has, in this case, periodic boundary conditions. Each site can either be empty or occupied by one vehicle having an integer velocity between *zero* and v_{max} ($v = 0, 1, ..., v_{max}$). At each discrete time step $t \to t + 1$ an arbitrary configuration of N vehicles is updated according to a set of four rules which are applied in parallel to all vehicles:

1. **Acceleration:** If the velocity v of a vehicle is less than v_{max} the velocity is increased by one $[v \to v + 1]$.

2. **Slowing down (braking):** If the distance d to the next vehicle ahead is less than the new velocity, the velocity is reduced to $d - 1$ $[v \to d - 1]$.

3. **Randomisation:** With probability p the velocity of a vehicle (if greater than zero) is reduced by one $[v \to v - 1]$.

4. **Vehicle motion:** Each vehicle advances a number of steps equal to its velocity v.

These four steps simulate general properties of single lane traffic and already demonstrate realistic behaviour. Step 1 assumes that a vehicle wants to travel at the maximum possible velocity, v_{max}. The acceleration is equal to 1. Step 2 is a deceleration step which ensures that vehicles do not crash into each other. The randomisation element introduced in step 3 is crucial in modelling traffic flow as it allows the evolution of realistic traffic scenarios. A vehicle may randomly decelerate by 1 for a given probability p. It accounts for the natural velocity fluctuations due to human behaviour. For example, the driver of a vehicle may overreact when braking, be cautious and leave a large distance between their vehicle and the vehicle in front, or simply be distracted. Without this step the model would be completely deterministic, every arrangement of vehicles and their corresponding velocities soon reaches a stationary pattern which is shifted backwards (i.e. opposite the direction of vehicle motion) one site per time step [5]. After the first three steps, the velocity is updated and the vehicles advance along the road, this is essentially a time step. An illustration of the steps can be found in Appendix A.

2.2. Investigating the single lane model

The following simulations have been modelled using Python. Initially, N vehicles are distributed randomly on a circular road of length L. The initial velocity of the vehicles is zero and the velocity can increase up to $v_{max} = 5$. Fig.1 illustrates a space-time diagram with 60 vehicles distributed over a road of length $L = 200$. By carefully examining this diagram, one can see that the vehicles move to the right (or remain stationary) with each time step. The dark clusters of vehicles (traffic jams) randomly form as a result of the velocity-fluctuations of the vehicles. A car approaching a traffic jam has to reduce its velocity and ultimately stop. It then remains stationary or experiences some slow advancement for a certain length of time until it leaves the traffic jam and can accelerate as normal. The clusters of congested vehicles propagate in the opposite direction to the vehicles direction of travel, a phenomenon resulting from the fact that a vehicle is only affected by the traffic in front (rule 2 in the NaSch model) and not the traffic behind.

For a circular road (having periodic boundary conditions) the total number of vehicles, N, does not change. As such, the density can be defined as:

$$\rho = \frac{N}{L} = \frac{\text{Number of vehicles occupying road}}{\text{Length of road}}. \tag{1}$$

For fundamental diagrams (flow vs. density) to be produced, it is also necessary we define the flow:

$$q = \frac{\text{Number of vehicles passing a point}}{\text{Number of time steps}}. \tag{2}$$

As the initial distribution of the vehicles is random and they all have zero initial velocity, it is important to omit at least the first few iterations when calculating the flow, to allow the pattern to settle. In our model, we omitted the first 10% of iterations when taking data. We did this for all the fundamental diagrams in this report. Fig.2a

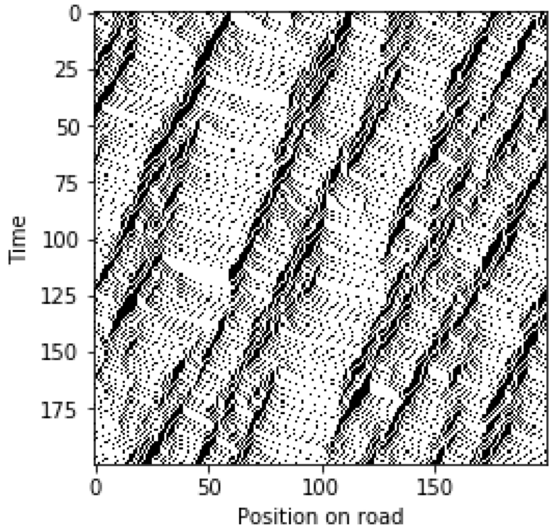


Figure 1: A space-time diagram for a circular road in the NaSch model with road length $L = 200$, density $\rho = 0.3$, probability velocity decreases $p = 0.3$, $v_{max} = 5$, over 200 iterations. Vehicles, indicated by darkened cells, propagate to the right in time. White space indicates the absence of a vehicle.

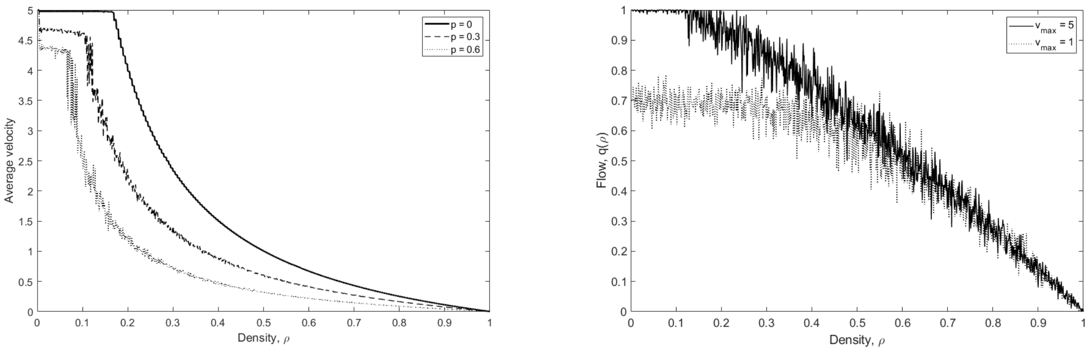
shows a plot of average velocity vs. density, varying p. The average velocity drops off quickly at a critical density. It can be seen that for $p > 0$ the average velocity is less than v_{max} even at very low densities, as expected.

Fig.2b shows a fundamental density-flow diagram in the NaSch model, varying v_{max}. This plot is not as expected, and differs from similar published fundamental diagrams. The reason for this difference is not fully understood, though we expect it must be related to how flow is defined in our model. We have thus far been unable to identify exactly how it differs. For high densities the plot is as expected, as $\rho \to 1$, $q(\rho) \to 0$ the road is so saturated with vehicles they can barely move. In previously published diagrams, $q(\rho) \to 0$ as $\rho \to 0$. That is, there are so few vehicles occupying the road that flow is almost non-existent. A peak flow is also seen for $\rho \approx 0.1$, a feature not emergent in our plots. The motivation for plotting $v_{max} = 1$ was to demonstrate symmetry about $\rho = 0.5$ [6], which we were unable to achieve as the flow does not tend to zero at low densities.

3. Applications

3.1. Buses

In this case, we take the vehicles in the model to be buses. There are two variable probabilities that are required to make the bus stops work: p_{stop} (the probability that a bus will stop at a bus stop) and p_{wait} (the probability that a bus will remain stationary

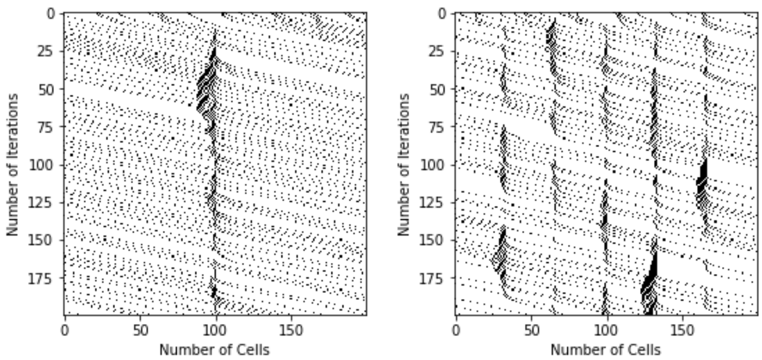


(a) Average velocity vs. density, ρ (b) Flow, q vs. density, ρ

Figure 2: These plots are for a road of length $L = 200$ over 200 iterations, increasing density in increments of $\delta\rho = 0.001$ from 0 to 1. In (a), $v_{max} = 5$ and plots are for three values of p; in (b), $p = 0.3$ and flow for two values of v_{max} are plotted.

once at a bus stop). The buses will still move according to the regular NaSch model, but the following rules are also applied:

1. Deceleration for a bus stop: if the bus at position i is in range of the bus stop at position j, with probability p_{stop} decrease the speed so that it stops at the bus stop $[v \to j - i]$.
2. Passengers boarding/alighting: if the bus is at the bus stop, it will remain at the bus stop with probability p_{wait} $[v = 0]$.



(a) Single stop (b) Five stops

Figure 3: A space-time diagram for a circular road in the NaSch model with 20 buses occupying a road of length $L = 200$ over 200 iterations. Probability velocity decreases $p = 0.2$, $v_{max} = 5$, $p_{stop} = 0.9$, $p_{wait} = 0.4$.

There are some limitations to this model. Firstly, a bus can stop at the bus stop without slowing in advance due to the first rule. This can happen if the bus reaches a bus stop site and then sets its velocity to zero according to the second rule introduced for bus stops. This in effect is as if the bus has no passengers that want to get off, and also no-one waiting at the bus stop, but the bus decides to stop anyway. The second issue is that the outcome of the first rule can change between iterations. As there is no uniqueness to a bus, on each iteration the bus will go through the first rule and decide whether it wants to stop due to p_{stop}. So, the only space that truly matters is the space just before the bus stop. Effectively, this is as though the bus decides to stop, be it to let passengers on or off, and then decides to drive past as it approaches the stop. Examples of space-time diagrams produced in this model are shown in Fig.3.

The clumping (or bunching) of buses occurs when buses are unable to keep to their schedule, which can happen for a number of reasons. A bus running behind schedule may have to take on extra passengers, as well as its normal load, because there will be passengers waiting at stops who would otherwise be serviced by the next scheduled bus. This additional load can result in the bus being even further behind schedule, creating a positive-feedback loop that perpetuates the situation. The next scheduled bus consequently has fewer passengers and can potentially run ahead of schedule. In time, these two buses, one being late and one being ahead of schedule, can bunch up. The effect being that two buses arrive at bus stops simultaneously, between there scheduled times. Other causes are that one bus may overtake another, or a bus may skip stops.

In our model, we account for the arrival and departure rates of passengers using the variable probability p_{wait} of the bus remaining at the stop once stationary. When p_{wait} is lowered, the number of clumped buses is reduced, meaning that a reduction of arrival/departure rate acts directly to decrease the frequency of clumping. The frequency of buses could also be altered by changing the density. At lower densities it was observed that clumping was minimised.

In reality, measures to reduce boarding time of passengers could be taken to improve the flow of buses. For example, a pre-paid fare such as a bus-pass could be used, eliminating the time taken for each passenger to purchase they ticket, one at a time. One strategy for reducing the clumping of buses, proposed in 2009 [7], is to use GPS data to directly impede the causes of bunching by allowing buses to communicate with one another to negotiate their speeds based on relative position.

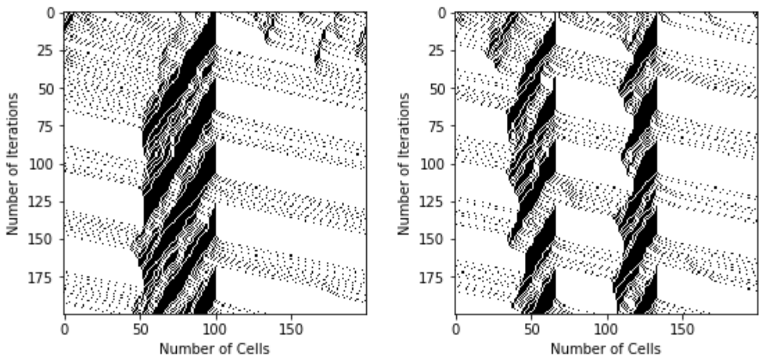
3.2. Traffic lights

The most important part of the traffic light code is the timing element. We used a sine function as it allowed us to introduce the periodic nature of traffic lights. This also allowed us to modify the period using two variables, omega, ω and t_{on}. Omega is a frequency defined as $\omega = (2\pi)/T$ so changing ω explicitly changes the period (T) with an inverse proportionality. Whereas t_{on} is a threshold variable used to allow the period of time the light is red to be different from the period the light is green. This code runs in two separate configurations: with one traffic light and with two traffic lights a distance $L/3$ apart. For each traffic light, the following rule is applied:

1. Deceleration for a traffic light: if the vehicle at position i is in range of a traffic

light on red at position j, decrease the velocity so that it stops at the traffic light $[v \rightarrow j - i]$.

The vehicles still move according to the regular NaSch model, so that if the traffic light is off (green) the NaSch model is restored. Examples of space-time diagrams produced in this case are shown in Fig.4.



(a) Single traffic light at position $L/2$

(b) Two traffic lights at intervals of $L/3$

Figure 4: A space-time diagram for a circular road in the NaSch model with 40 vehicles occupying a road of length $L = 200$ over 200 iterations. Probability velocity decreases $p = 0.3$, $v_{max} = 5$, $\omega = 0.15$, $t_{on} = 0$.

The limitation of this model is that the timing of the traffic light is not the most realistic. At most traffic light junctions there are calculations done to decide how long a traffic light should be on to minimise congestion. For a given traffic flow, outlined in Fig.4, to calculate the optimum time intervals between red and green lights, it is required to find the values of ω and t_{on} which maximise flow and average velocity on the fundamental diagrams. It was found that the most optimal values were $\omega = 0.2$, $t_{on} = 0.6$ for a single traffic light and $\omega = 0.3$, $t_{on} = 0.7$ for two traffic lights. This produces plots that have short on-off periods for the traffic lights, which allows for higher flows whilst breaking up congestion. For example, Fig.4 is an example where the high on-off periods for the traffic lights impedes flow by causing a jam behind the traffic lights.

3.3. Roundabout

The roundabout is defined by the waiting position, $wait_{pos}$, the exit position, $exit_{pos}$, and the entry position, ent_{pos}. The $wait_{pos}$ is the site at which a vehicles waits for the roundabout to be empty so that it can enter. The $exit_{pos}$ is the site at which a vehicle on the roundabout can exit if it wishes to. The ent_{pos} is the site at which vehicles enter the circular road. This describes a roundabout with an entrance *from* the circular road and an exit *to* the circular road, as well as a new two-lane road with exit and entrance sites at the roundabout (see Fig.5). There are also variable probabilities for exiting

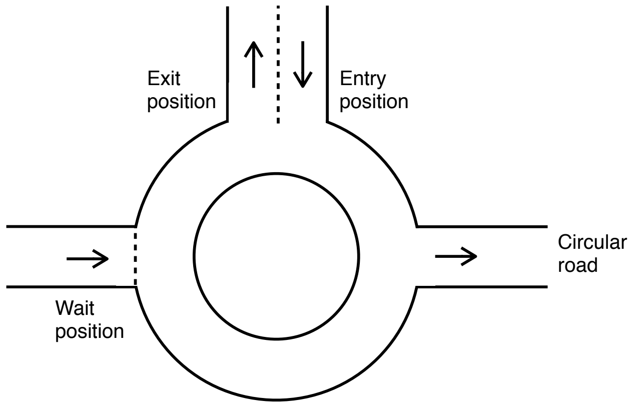


Figure 5: Structure of the roundabout used in our model. The arrows indicate the direction of vehicle travel.

and entering the roundabout (p_{exit} and p_{ent} respectively) from the two-lane road, which can be used to moderate the flow of traffic into and out of the circular road. For the roundabout, the vehicles will still move according to the regular NaSch model, but the following rules are also applied:

1. Deceleration: if the vehicle at position i is in range of the roundabout, decrease its velocity so that it stops at the waiting position $[v \to wait_{pos} - i]$.

2. Waiting to enter the roundabout: if there is a vehicle on the roundabout, remain stationary at the waiting position $[v = 0]$.

3. Entering the roundabout from the circular road: if there are no vehicles on the roundabout, enter the roundabout and move as normal $[v = 1]$.

4. Exiting the roundabout: if the vehicle is at the exit position, with probability p_{exit} remove it from the circular road.

5. Entering the roundabout from the two-lane road: if there are no vehicles on the roundabout, with probability p_{ent} enter the roundabout and move as normal $[v = 1]$.

There are some limitations to this model. Firstly, a vehicle that enters the roundabout from the connected two-lane road cannot then choose to go around the roundabout and exit back onto the two-lane road, as this violates the flow of the circular road. A vehicle that would want to exit would have to go all the way around the circular road and back to the other side to re-join the roundabout and exit. The second issue is that you can only have one vehicle on this roundabout at once, but in the real world you would be able to see where a car is going and be able to enter the roundabout if you know that you will not collide with them. In the real world, there are also rules for giving the

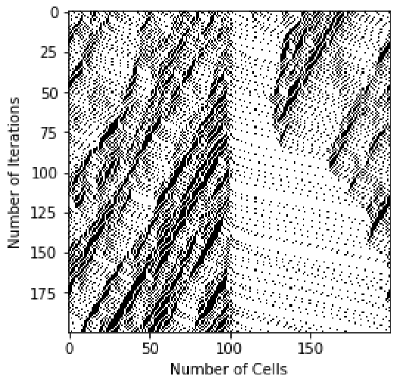


Figure 6: A space-time diagram for the roundabout scenario for a road of length $L = 200$ over 200 iterations: $p_{exit} = 0.6$, $p_{ent} = 0.5$, $p = 0.3$, $v_{max} = 5$, $\rho = 0.3$.

right of way which are not reproduced in this model. Our model is similar to that of a mini-roundabout with controlled entrance conditions. Fig.6 illustrates a space-time diagram for a road of length L = 200 over 200 iterations for the roundabout model.

In essence, roundabouts and traffic lights function with similar principles: reducing traffic flow to allow the break up of congestion. At low densities a traffic light and a roundabout produce similar flows and average velocities, so on roads with low car frequency, such as B roads, they can be used interchangeably. However at higher densities the flow rate is significantly reduced on roundabouts compared to traffic lights. This occurs for roundabouts because in the model only one car can be on the roundabout at once, which leads to congestion at the wait position when the density is high, whereas a traffic light allows several cars to leave within each red-green period. Realistically flow is slightly higher due to multiple cars being able to travel on roundabouts at once, yet not significantly higher to allow them to replace traffic lights at higher densities.

4. Conclusion

The fundamental diagram outlined in section 2 allowed us to analyse the flows of the different models adapted from the Nagel-Schreckenberg model. Bus clumping was produced for varying traffic flows of differing density, which arose due to the high frequency of buses stopping paired with the irregularity of buses waiting for passengers to board/disembark. For a given traffic flow, outlined in Fig.4, it was found that to produce optimum time intervals between red and green lights the variables were set as: $\omega = 0.2$, $t_{on} = 0.6$ for a single traffic light and $\omega = 0.3$, $t_{on} = 0.7$ for two traffic lights. At lower traffic flows, a roundabout is a suitable substitute for traffic lights. As the car density increases, a roundabout becomes insufficient to deal with these higher densities leading to high congestion levels.

Although the models have their flaws, several of the rules and assumptions made do give a representation of real traffic. The allowance of randomised deceleration was an example of where real traffic was taken into consideration. Another such example was the second rule of the basic NaSch model. An alternative method for this rule would be to allow a vehicle at position i to decrease its velocity to 1 less than the velocity of a vehicle at position $i+1$ to prevent a collision. This modified rule might be more realistic, as in most cases it is likely that a driver would see the car in front decelerate and would reduce their velocity to compensate, rather than suddenly braking hard on approach. Additionally, allowing cars/buses to slow down on approach to a traffic light, roundabout or bus stop, rather than suddenly stopping, is another example of real vehicle behaviour, as again a driver would identify the hazard and gradually brake.

One modication that could be investigated is the construction of the first rule for traffic lights. This only allows a traffic light to be on or off. Real traffic lights have an amber phase to prevent sudden braking and including this in the model would create a more realistic simulation. Another could be adding in a second two lane road for the roundabout, as this is more commonly seen in the real world as a result to deal with multiple lanes meeting. Then different flow rates could be applied to the different roads to simulate roads of various car frequency meeting.

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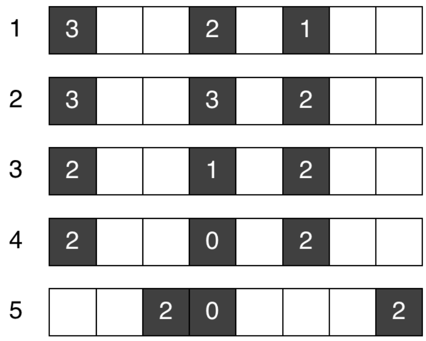


Figure 7: Step-by-step illustration of a single update in the NaSch model. Darkened cells are occupied by vehicles and their integer velocities are shown.

A. An example of the NaSch rule set

To properly understand this report it is essential to have a good grasp of the NaSch model rule set and the steps taken in each iteration. An example of a single iteration is given in Fig.7 for $v_{max} = 3$ and $L = 8$.

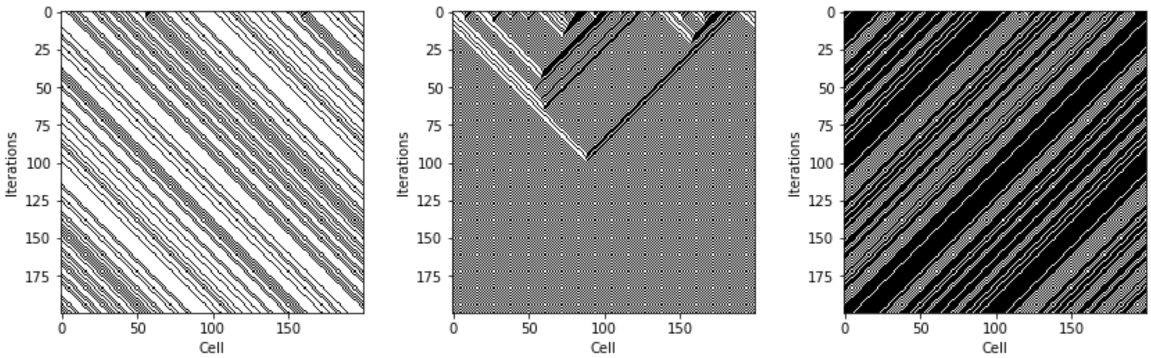
1. Initial configuration.
2. Acceleration: vehicles at sites 4 and 6 accelerate by 1 ($v_{max} = 3$).
3. Braking: vehicles at sites 1 and 4 need to brake.
4. Randomisation ($p = 0.3$): the vehicle occupying site 4 reduces its velocity by 1.
5. Motion: the vehicles advance a number of sites equal to their new velocity.

B. Rule 184

Rule 184 (also known as the "traffic rule") is a one-dimensional binary cellular automaton rule. It is a simple model for traffic flow on a single-lane road, and forms the basis of many CA models [8] including the Nagel-Schreckenberg model [5]. A one-dimensional array of cells, each having a binary value (0 or 1), evolves as the following rule is applied to each cell in the array simultaneously:

Initial pattern	111	110	101	100	011	010	001	000
New centre cell	1	0	1	1	1	0	0	0

This table defines the updated state of each cell as a function of the previous state and values of neighbouring cells. The bottom row, 10111000, when viewed as a binary number is equal to the decimal number 184, hence the name of the rule. A significant property of the dynamics of the system is that for a set of cells with periodic boundary conditions, the number of 1s (and hence the number of 0s) remains invariant in the system's evolution. If each cell of value 1 is taken to contain a particle, under Rule 184 these particles behave similarly to vehicles in single lane traffic. Consequently we can produce space-time diagrams of Rule 184 for different densities using the NaSch simulation utilised in this report. Using $v_{max} = 1$ and $p = 0$ previously published diagrams [9, 10] can be reproduced for densities $\rho = 0.25, 0.5$ and 0.75. The space-time diagrams in Fig.8 produced from the model used in this report are in agreement with what is expected from Rule 184. This confirms that the rule set used in this project has been used properly.



(a) $\rho = 0.25$ (b) $\rho = 0.5$ (c) $\rho = 0.75$

Figure 8: Rule 184 for three different densities, over 200 iterations for a random initial configuration of occupied cells.